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A SEMIEMPIRICAL TEST FOR DYNAMICAL INSTABILITY IN LUMINOUS BLUE VARIABLES

RICHARD B. STOTHERS
Institute for Space Studies, NASA/Goddard Space Flight Center, 2880 Broadway, New York, NY 10025
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ABSTRACT

By employing as input parameters only observationally determined stellar surface quantities, detailed envelope models for six well-observed luminous blue variables at quiescence are constructed and tested for marginal dynamical instability. The best-observed of these objects, P Cyg, proves to be exactly in a state of marginal dynamical instability, just as predicted. Within the larger observational errors estimated for the five other objects, they too are probably in the same marginally unstable state as P Cyg.

Subject headings: stars: individual (P Cygni) — stars: oscillations — stars: variables: other (luminous blue variables)

1. INTRODUCTION

The origin of the S Doradus–type instability in the stars known as luminous blue variables (LBVs) is still a great mystery (Nota & Lamers 1997). Many of the published efforts to solve this riddle have attempted to establish the probable locations of LBVs along computed evolutionary tracks. This approach, by itself, is obviously incomplete and cannot be conclusive. Alternatively, a potential instability mechanism might be applied to a model of a particular star instead of to a generic stellar model. As in the case of conventional pulsating variable stars, one might proceed by adopting the observed surface parameters for a particular star in order to construct a theoretical envelope model, without any need for knowledge of the star's previous evolutionary history. In the case of several classical LBVs that are spectroscopically well observed, there are now available reliable surface gravities, from which stellar masses can be derived. No unknown values of the fundamental surface parameters still remain that must be arbitrarily assigned.

In the present paper, we analyze six well-observed LBVs from a theoretical (or, more correctly, a semiempirical) point of view. We ask whether any or all of them could be dynamically unstable, within a small tolerance allowed by the errors of observation. One proposed scenario (Stothers & Chin 1996) predicts that LBVs at quiescence lie very close to dynamical instability and that crossing the threshold of instability leads to their S Doradus–like outbursts.

2. OBSERVATIONAL DATA

The six LBVs to be tested are listed in Table 1. References provided there pertain only to the observed masses (M), luminosities (L), and effective temperatures (T_e), whose possible errors will be discussed below. Assigned metallicities (Z) are those typical of very young stars in the host galaxies (Kilian 1992; Rolleston 1995; Luck et al. 1998). Possible errors in Z are not likely to exceed 0.2 dex since observations of the six LBVs indicate that their metallicities appear to be normal for their galactic environments.

Hydrogen abundances have been measured for a number of LBVs and LBV candidates. We list the derived abundances here: $X = 0.33$ (Barlow 1991), 0.38 (Langer et al. 1994), or 0.44 (Lamers et al. 1996) for P Cyg; $X = 0.36$ (Lennon et al. 1994) for R71; $X = 0.49$ (Barlow 1991), 0.37 (Smith, Crowther, & Prinja 1994), or 0.18 (Leitherer et al. 1994) for AG Car; $X = 0.33$ (Smith et al. 1994) for He 3-519; $X = 0.37$ (Schmutz et al. 1991) or 0.49 (Pasquali et al. 1997a) for R84; $X = 0.33$ (Crowther 1997) for R127; $X = 0.32$ (Crowther & Smith 1997; Pasquali et al. 1997a) for BE294; and $X = 0.33$ (Pasquali et al. 1997b) for HDE 269445. The average of all these determinations is $\langle X \rangle = 0.36$, with a dispersion that is surprisingly small. We might have expected to find far more scatter in such hydrogen-depleted stellar remnants having such a wide range of masses. Moreover, three evolutionary tracks covering the range of most LBV luminosities, and extending all the way to the threshold of ionization-induced dynamical instability in the hydrogen-poor envelopes, have predicted that $X = 0.22$, 0.20, and 0.12, or $\langle X \rangle = 0.18$, albeit with considerable uncertainty (Stothers & Chin 1996). To cover the various contingencies, we shall adopt two values for X: the currently accepted one, $X = 0.35$, and half that value, $X = 0.175$.

3. THEORETICAL ASSUMPTIONS

Recent linear and nonlinear hydrodynamical calculations to look for dynamical instability in highly nonadiabatic stellar envelopes have demonstrated explicitly that dynamical instability occurs when σ^2, the square of the fundamental eigenfrequency for small *adiabatic* radial displacements, falls below zero (Stothers 1999b). (Nonadiabatic effects are manifested through the envelope's pulsational and thermal behavior.) Following this demonstration and its corroboration of our earlier theoretical work based on assuming $\sigma^2 \leq 0$ as the appropriate criterion, we shall evaluate σ^2 for realistic equilibrium models of the envelopes of the six stars in Table 1.

Specifically, we adopt the tabulated empirical values of M, L, T_e, Z, and X for each star, and then we integrate, step by step, from near the top of the atmosphere down into the deep interior, a detailed model of the stellar envelope. The Saha ionization equation is used to evaluate state quantities; opacities are taken from Iglesias, Rogers, & Wilson (1992); and convection is included by using standard mixing-length theory, with a ratio of mixing length to local pressure scale height, α_P, equal to 1.4. Thus, the input physics is identical to that employed in our previous recent work on LBVs.

To determine σ^2, we integrate the linear adiabatic radial wave equation that governs dynamical instability (Ledoux 1958, eq. [12.12]) from the surface down to a layer where the displacement amplitude becomes negligible. For convenience, our results for σ^2 are expressed in terms of a non-

TABLE 1

TEST RESULTS FOR DYNAMICAL INSTABILITY IN SIX LUMINOUS BLUE VARIABLES

							$X = 0.350$		$X = 0.175$		
VARIABLE	GALAXY	$M/M_\odot$	$\log (L/L_\odot)$	$\log T_e$	$\log \Lambda$	Z	ω^2	$\delta \log \Lambda$	ω^2	$\delta \log \Lambda$	REFERENCES
P Cyg	MW	23	5.86	4.27	4.50	0.030	0.00	0.00	0.22	0.05	1, 2
HD 160529	MW	13	5.46	4.04	4.35	0.030	0.71	0.09	1.02	0.13	1, 3
S Dor	LMC	23	5.82	4.35	4.46	0.010	1.10	0.12	2.01	0.17	1, 4
R71	LMC	20	5.85	4.13	4.55	0.010	−0.02	−0.01	0.35	0.04	5
R110	LMC	10	5.46	4.01	4.46	0.010	0.14	0.03	0.39	0.07	1, 6
R40	SMC	16	5.64	4.00	4.44	0.004	1.03	0.11	1.20	0.15	7

REFERENCES.—(1) Humphreys & Davidson 1994; (2) Pauldrach & Puls 1990; (3) Sterken et al. 1991; (4) Wolf & Stahl 1990; (5) Lennon et al. 1994; (6) Stahl et al. 1990; (7) Szeifert et al. 1993.

dimensional eigenvalue, $\omega^2 = \sigma^2 R^3/GM$. We also define a normalized luminosity-to-mass ratio, $\Lambda = (L/L_\odot)(M/M_\odot)^{-1}$.

4. RESULTS OF THE CALCULATIONS

Results of our calculations for dynamical instability are presented in Table 1 under the headings of our two adopted values of X. Notice how close to the critical value $\omega^2 = 0$ all six stars lie, in comparison with a normal blue supergiant that has evolved with little mass loss and consequently has $\omega^2 \approx 7$. Based on purely observational data, two of the stars turn out to be formally slightly dynamically unstable, and the four others could be too, if possible errors of observation are taken into account.

Consider first the best-observed object, the prototype LBV, P Cyg. This particular case is crucial, because we have shown elsewhere that P Cyg's measured rate of decline of effective temperature, $d(\log T_e)/dt = -0.027 \pm 0.004$ century^{-1} (Lamers & de Groot 1992), is accurately reproduced by our robustly predicted rate, $d(\log T_e)/dt = -0.028 \pm 0.003$ century^{-1}, based on stellar evolutionary models undergoing slow secular cycles that are triggered by dynamical instability in the outer envelope (Stothers & Chin 1995). With all of its surface parameters accepted as given, P Cyg proves to be marginally dynamically unstable, $\omega^2 = 0.00$, just as we would have predicted from its present state of precarious quiescence after its last outburst in the seventeenth century. Varying the parameters of P Cyg within their estimated uncertainties (Table 2) shows that possible errors in T_e, Z, and α_P have virtually no effect on ω^2. On the other hand, small plausible reductions (increments) of Λ and X exert small stabilizing (destabilizing) influences.

This sensitivity can be understood in a qualitative way as follows. The outer envelope of a LBV contains a negligible mass and a very high radiation pressure compared to gas pressure. For such a star,

$$\omega^2 \approx (5/2)(3\Gamma_1 - 4) , \tag{1}$$

where $\Gamma_1 \approx (4/3) + (\beta/6)$ and $\beta = P_{\rm gas}/(P_{\rm gas} + P_{\rm rad})$ under the simplifying assumption of a fully ionized gas at all layers (Stothers 1999a). Assuming also a constant (electron–scattering) opacity, $\kappa = 0.20(1 + X)$ cm^2 g^{-1}, the equation of radiative equilibrium can be integrated to give

$$1 - \beta = \kappa L/(4\pi c G M) . \tag{2}$$

Equation (1) then becomes

$$\omega^2 \approx \frac{5}{4}\left(1 - \frac{\kappa \Lambda L_\odot}{4\pi c G M_\odot}\right) . \tag{3}$$

Differentially,

$$\delta\omega^2 \approx -\frac{5 \ln 10}{4}\, \delta \log \Lambda , \quad \delta\omega^2 \approx -\frac{5}{4}\frac{\delta X}{1 + X} . \tag{4}$$

Accordingly, as Λ or X decreases (increases), ω^2 grows (diminishes), thus tending to stabilize (destabilize) the envelope. Note that ω^2 depends on L and M only through their ratio, Λ. This expression for ω^2 is not quantitatively accurate, however, because it does not include the effects of partial ionization of the gas, so that Γ_1 cannot fall below 4/3 and therefore ω^2 can never become negative.

Returning to Table 1, we now ask what changes in Λ would be needed for the five other stars listed there to be exactly in a state of marginal dynamical instability. These changes, $\delta \log \Lambda$, are also entered in the table. As anticipated, they are quite small, in all cases being less than 0.13 for $X = 0.35$ and less than 0.18 for $X = 0.175$.

Estimates of possible errors affecting the observed masses and luminosities that combine into Λ are not always available. For P Cyg (Pauldrach & Puls 1990), however, the possible internal error of $\log \Lambda$ can be estimated as ± 0.04. Since in general the observed masses are obtained from a combination of surface gravities (g), luminosities, and effective temperatures, any error in $\log \Lambda$ amounts to

$$\delta \log \Lambda = 4\delta \log T_e - \delta \log g . \tag{5}$$

Systematic errors in $\log g$ for most LBVs may be fairly large, because the atmospheres of LBVs approach the Eddington luminosity limit (e.g., Wolf & Stahl 1990). Furthermore, the measured effective temperatures for several LBVs may also be in error by up to $\sim$10% (Humphreys & Davidson 1994). Total possible errors in $\log \Lambda$ could, therefore, easily be as much as ± 0.15 for some stars that are not so well observed as P Cyg. Interestingly, the only other star favored with relatively small measurement errors is R71

TABLE 2

TEST RESULTS FOR DYNAMICAL INSTABILITY IN P CYG

$M/M_\odot$	$\log (L/L_\odot)$	$\log T_e$	X	Z	α_P	ω^2
23	5.86	4.27	0.350	0.03	1.4	0.00
25	5.86	4.27	0.350	0.03	1.4	0.16
23	*5.82*	4.27	0.350	0.03	1.4	0.16
23	5.86	*4.30*	0.350	0.03	1.4	0.01
23	5.86	4.27	*0.175*	0.03	1.4	0.22
23	5.86	4.27	0.350	*0.02*	1.4	0.08
23	5.86	4.27	0.350	0.03	*2.0*	0.02

(Lennon et al. 1994), which Table 1 reveals, in fact, to be slightly dynamically unstable.

5. CONCLUSION

We conclude that all six LBVs in Table 1 lie well within the realm of possible dynamical instability. Since all these stars seem to be, in any case, formally very close to this state, it is plausible to assume that all have actually arrived there or are at least hovering at its border.

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